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Solving generalized quintic complex Ginzburg–Landau equation by homotopy analysis method

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h-curve;
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Abstract In this paper, the generalized quintic complex Ginzburg–Landau equation is considered to be solved, by means of the homotopy analysis method (HAM). Two examples are solved to illustrate the efficiency of the proposed method. By plotting the h-curve of the examples, the region of convergence is determined.

1. Introduction

The cubic–quintic complex Ginzburg–Landau equation is a continuous approximation to the dynamics of the field in a passively mode-locked laser.It has also been proven to be useful in describing important phenomena such as ultrashort pulse propagation in optical transmission lines with spectral filtering and erbium-doped fiber amplifiers, the GCGL equation was used successfully in modeling of other nonequilibrium processes in physics, The quintic terms in the equation describe important physics, which is lacking in other models in the literature [3]. It arises in a variety of settings, including nonlinear optics, fluid dynamics, chemical physics, mathematical biology, condensed matter physics, and statistical mechanics. In fluid dynamics the GCGL is found, for example, in the study of Poiseuille flow, the nonlinear growth of convection rolls in the Rayleigh–Benard problem and Taylor–Couette flow. In this case, the bifurcation parameter R plays the role of a Reynolds number. The equation also arises in the study of chemical systems governed by reaction–diffusion equations. The GCGL equation plays the role of a simplified set of fluid dynamic equations.

The Ginzburg–Landau equation has been used to study many practical problems such as chemical turbulence, Poiseuille flow, Taylor–Coutte flow, Rayleigh–Benard convection, reaction–diffusion systems, nonlinear optics, and hydrodynamical stability problems. It exhibits rich dynamics and has become a paradigm for the transition to spatio-temporal chaos. Also some other details about the other different types of Ginzburg equation can be considered in [3,7,8,10,24,25,26,27,31,33,34].

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Table 1 The results of example 1 via the HAM at $x = 1$.

t	0	0.2	0.4	0.6	0.8	1
Error	0	1.1102e−016	1.1102e−016	1.4983e−016	1.4895e−015	4.7744e−014

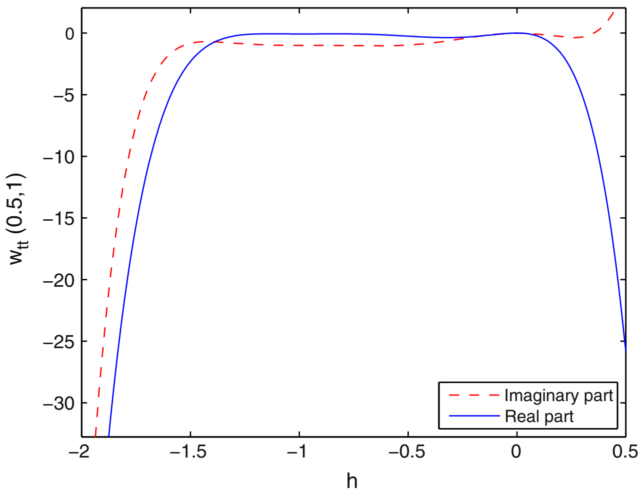

Fig. 1 The h-curve of $w_{tt}(0.5,1)$ given by 7-approximation of the HAM.

We consider generalized quintic complex Ginzburg–Landau equation with the form

$$\frac{\partial w}{\partial t} = (1+ia)\frac{\partial^2 w}{\partial x^2} + Rw - (1+ib)|w|^{2n}w - (1+id)|w|^{4n}w \; n \geqslant 1,$$
$$w(x,0) = f(x), \; i^2 = -1, \quad (1)$$

where $a,\ b,\ d,\ R$ are real constants and $w = w(x,t)$ is a complex unknown function and t is a nonnegative real value, x is a real value [3,9,13,14,29,33,34]. Wazwaz studied Eq. (1) by using the separation of variables method in [33].

In this work, the HAM is being considered to obtain the approximate solution of Eq. (1). This method is a powerful analytical method to solve the nonlinear problems and was first introduced and used by Liao [23]. Recently, this method has been well applied to solve many types of problems in science and engineering [1,2,4–6,11,12,17–19,28,30,32]. Homotopy analysis method contains an auxiliary parameter h which provides a simple way to adjust and control the convergent region and the rate of convergence of the series solution. HAM bears a very rapid convergence of the solution series in most cases, usually only a few iterations with satisfactory approximate solution, as well [20–23]. Total discription of this paper is as follows: In Section 2, some preliminaries are given, and in Section 3, the main idea of this paper is brought, and finally in Section 4, two examples are solved and h-curves are plotted to show the region of convergence.

2. Preliminaries

Let the following differential equation:

$$N[w(x,t)] = 0,$$

where N is a nonlinear operator, x and t denote the independent variables and w is an unknown function. Via the HAM, the zeroth-order deformation equation is:

$$(1-q)L[\Phi(x,t,q) - w_0(x,t)] = qhH(x,t)N[\Phi(x,t,q)], \quad (2)$$

where $q \in [0,1]$ is the embedding parameter, $h \neq 0$ is an auxiliary parameter, L is an auxiliary linear operator and $H(x,t)$ is an auxiliary function. $\Phi(x,t,q)$ is an unknown function and $w_0(x,t)$ is an initial guess of $w(x,t)$. It is clear, if $q = 0$ and $q = 1$ then:

$$\Phi(x,t,0) = w_0(x,t), \quad \Phi(x,t,1) = w(x,t),$$

respectively. Therefore, when q increases from 0 to 1, the solution $\Phi(x,t,q)$ varies from $w_0(x,t)$ to the exact solution $w(x,t)$. By Taylor's theorem, it can be expanded $\Phi(x,t,q)$ in a power series of the embedding parameter q as comes:

$$\Phi(x,t,q) = w_0(x,t) + \sum_{m=1}^{\infty} w_m(x,t)q^m, \quad (3)$$

where

$$w_m(x,t) = \frac{1}{m!}\frac{\partial^m \Phi(x,t,q)}{\partial q^m}\bigg|_{q=0}. \quad (4)$$

Let the initial guess $w_0(x,t)$, the auxiliaray linear operator L, the nonzero auxiliaray parameter h and the auxiliaray function $H(x,t)$ be properly chosen so that the power series Eq. (3) converges at $q = 1$, then, it can be seen:

$$w(x,t) = w_0(x,t) + \sum_{m=1}^{\infty} w_m(x,t), \quad (5)$$

which must be the solution of the original nonlinear equation. Now, we define the following set of vectors:

$$\vec{w}_n = \{w_0(x,t), w_1(x,t), \ldots, w_n(x,t)\}. \quad (6)$$

Table 2 The errors of the example 1 via HAM at (2, 0.7).

n	Approximation at (2,0.7)	by HAM ($h = -1$)	Error
4	−9.028807597814920e−001	+4.281032621672038e−001i	1.393798067426522e−003
8	−9.040720445762246e−001	+4.273799333574138e−001i	1.109812228348067e−007
12	−9.040721420156820e−001	+4.273798802345464e−001i	1.554304121895804e−012
16	−9.040721420170611e−001	+4.273798802338299e−001i	1.570092458683775e−016

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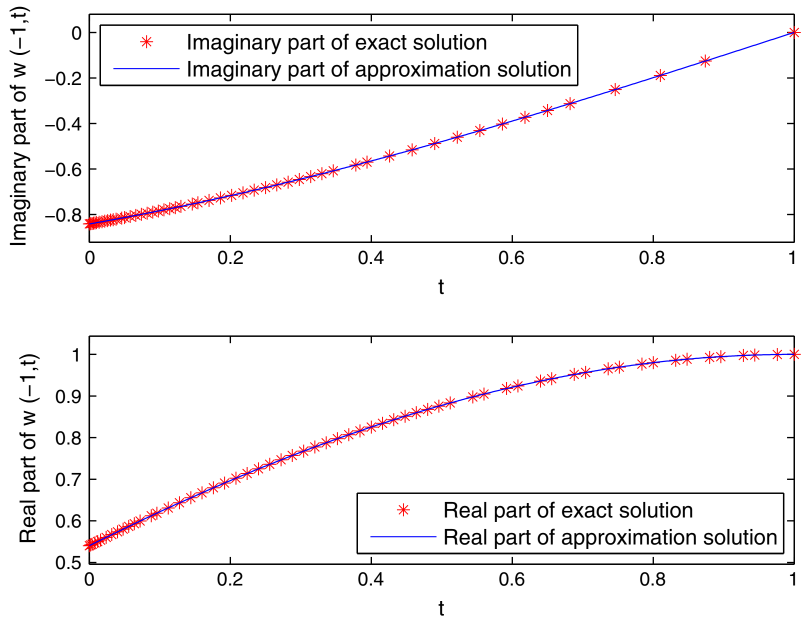

Fig. 2 Imaginary and real parts of exact and approximation solutions of Example 1 when $x = -1$ and $m = 7$.

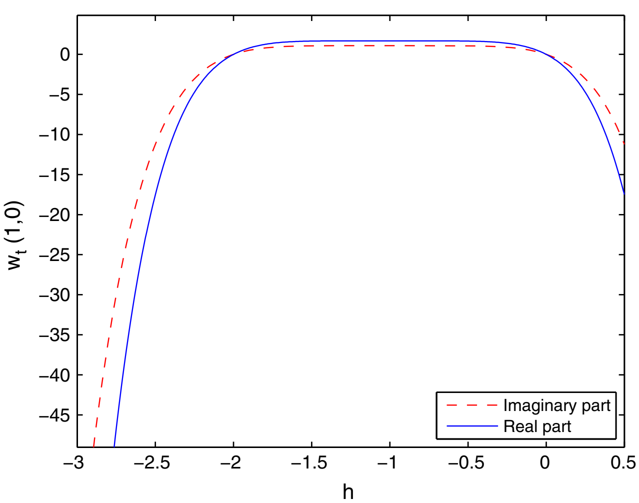

Fig. 3 The h-curve for $w_t(1,0)$ given by 6-approximation ($m = 6$).

By differentiating the zeroth order deformation Eq. (2) m times with respect to the embedding parameter q and then setting $q = 0$ and finally dividing by $m!$, we will have the following mth order deformation equation:

$$L[w_m(x,t) - \chi_m w_{m-1}(x,t)] = hH(x,t)R_m(\vec{w}_{m-1}), \tag{7}$$

where

$$R_m(\vec{w}_{m-1}) = \frac{1}{(m-1)!}\frac{\partial^{m-1}N[\Phi(x,t,q)]}{\partial q^{m-1}}|_{q=0}, \tag{8}$$

and

$$\chi_m = \begin{cases} 0 & m \leqslant 1, \\ 1 & m > 1. \end{cases} \tag{9}$$

It should be mentioned that $w_m(x,t)$ for $m \geqslant 1$ is governed by the linear Eq. (7) with linear boundary conditions that come from the original problem. For more details about the HAM, we refer the reader to [20,21].

3. Main idea

We consider Eq. (1) as follows:

$$\frac{\partial w}{\partial t} = (1+ia)\frac{\partial^2 w}{\partial x^2} + Rw - (1+ib)w^{n+1}\bar{w}^n - (1+id)w^{2n+1}\bar{w}^{2n}n \geqslant 1, \tag{10}$$

$w(x,0) = f(x), i^2 = -1,$

and

$$L[\Phi(x,t,q)] = \frac{\partial \Phi(x,t,q)}{\partial t}, \quad L(c) = 0, \tag{11}$$

where c is a real constant,

$$N[\Phi(x,t,q)] = \frac{\partial \Phi(x,t,q)}{\partial t} - (1+ia)\frac{\partial^2 \Phi(x,t,q)}{\partial x^2} - R\Phi(x,t,q) + (1+ib)\Phi^{n+1}(x,t,q)\bar{\Phi}^n(x,t,q) + (1+id)\Phi^{2n+1}(x,t,q)\bar{\Phi}^{2n}(x,t,q), \tag{12}$$

and $H(x,t) = 1$. The zeroth-order deformation equation is:

$$(1-q)L[\Phi(x,t,q) - w_0] = qhN[\Phi(x,t,q)]. \tag{13}$$

Also, the mth-order deformation equation:

$$L[w_m - \chi_m w_{m-1}] = hR_m(\vec{w}_{m-1}), \tag{14}$$

where

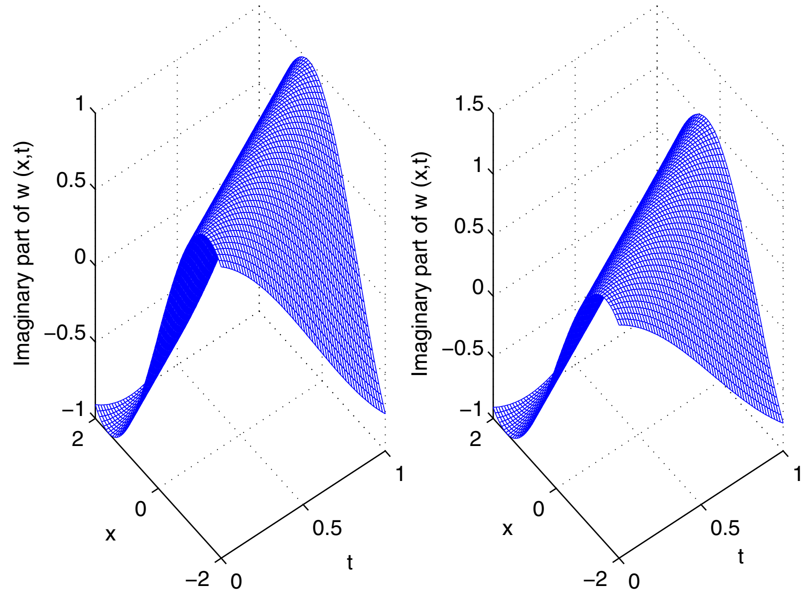

Fig. 4 Imaginary parts of the approximate and the exact(left) solutions of Example 2 when $m = 6$.

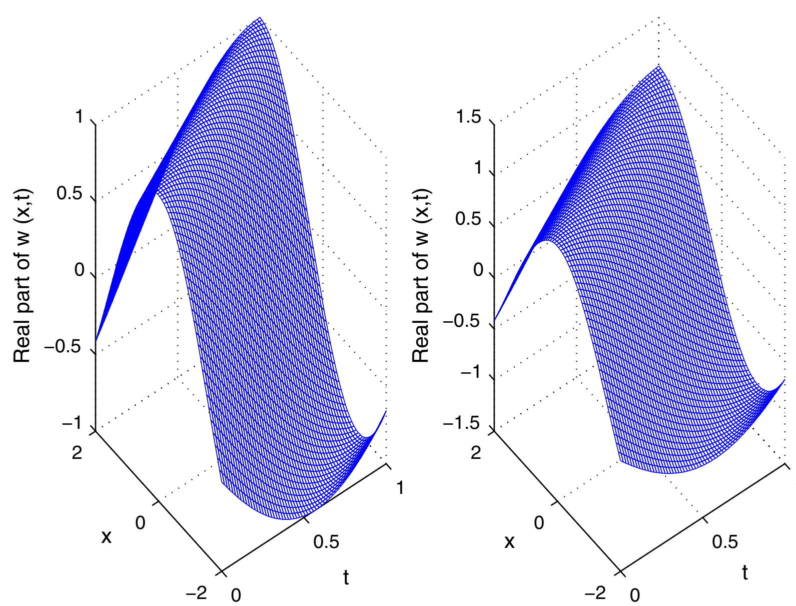

Fig. 5 Real parts of the approximate and the exact(left) solutions of Example 2 when $m = 6$.

$$R_m(\vec{w}_{m-1}) = \frac{\partial w_{m-1}}{\partial t} - (1+ia)\frac{\partial^2 w_{m-1}}{\partial x^2} - Rw_{m-1} + (1+ib)\sum_{k_1=0}^{m-1}\sum_{k_2=0}^{k_1}\sum_{k_3=0}^{k_1-k_2}\cdots\sum_{k_{n+1}=0}^{k_1-k_2-\cdots-k_n}\sum_{\alpha_1=0}^{m-1-k_1}\sum_{\alpha_2=0}^{m-1-k_1-\alpha_1}\cdots\sum_{\alpha_{n-1}=0}^{m-1-k_1-\alpha_1-\cdots-\alpha_{n-2}} w_{k_2},$$
$$w_{k_3}\cdots w_{k_{n+1}} w_{(k_1-k_2-\cdots-k_{n+1})}\bar{w}_{\alpha_1}\bar{w}_{\alpha_2}\cdots\bar{w}_{\alpha_{n-1}}\bar{w}_{(m-1-k_1-\alpha_1-\cdots-\alpha_{n-1})} + (1+id)\sum_{k_1=0}^{m-1}\sum_{k_2=0}^{k_1}\sum_{k_3=0}^{k_1-k_2}\cdots\sum_{k_{2n+1}=0}^{k_1-k_2-\cdots-k_{2n}}\sum_{\alpha_1=0}^{m-1-k_1}\sum_{\alpha_2=0}^{m-1-k_1-\alpha_1}\cdots\sum_{\alpha_{2n-1}=0}^{(m-1-k_1-\alpha_1-\cdots-\alpha_{2n-2})} w_{k_2},$$
$$w_{k_3}\cdots w_{k_{2n+1}} w_{(k_1-k_2-\cdots-k_{2n+1})}\bar{w}_{\alpha_1}\bar{w}_{\alpha_2}\cdots\bar{w}_{\alpha_{2n-1}}\bar{w}_{(m-1-k_1-\alpha_1-\cdots-\alpha_{2n-1})}. \tag{15}$$

So,

$$w_m = \chi_m w_{m-1} + h\int_0^t R_m(\vec{w}_{m-1})dt + c, \quad m \geqslant 1. \tag{16}$$

Now, in order to solve Eq. (10), with other semi-analytical method, we consider the variational iteration method (VIM) [15,16] to compare with the HAM. Let the following nonlinear differential equation is given:

$$Lw(x,t) + Nw(x,t) = g(x,t),$$

where L and N are linear and nonlinear operators respectively, and g is inhomogeneous term. The correction functional can be written as follows:

$$w_{m+1}(x,t) = w_m(x,t) + \int_0^t \lambda(\xi)(Lw_m(\xi) + N\tilde{w}_m(\xi) - g(\xi))d\xi,$$
$$m \geqslant 0,$$

where λ is general Lagrange multiplier which can be calculated by variational theory, that it means $\delta\tilde{w}_m = 0$, and the exact solution is determined by $w = \lim_{m\to\infty} w_m$ [15,33]. By applying the VIM to solve Eq. (10), the correction functional of Eq. (10) can be considered as follows:

$$w_{m+1} = w_m + \int_0^t \lambda(\xi)\left(\frac{\partial w_m}{\partial \xi}(x,\xi) - (1+ia)\frac{\partial^2 \tilde{w}_m}{\partial x^2}(x,\xi)\right.$$
$$\left. - R\tilde{w}_m(x,\xi) + (1+ib)\tilde{w}_m^{n+1}\tilde{\bar{w}}_m^{n}(x,\xi) + (1+id)\tilde{w}_m^{2n+1}\tilde{\bar{w}}_m^{2n}(x,\xi)\right)d\xi. \tag{17}$$

To make this correction functional stationary, by applying, $\delta w_m(x,0) = 0$, we obtain:

$$\delta w_{m+1}(x,t) = \delta w_m(x,t) + \int_0^t \lambda(\xi)\left(\frac{\partial}{\partial \xi}\delta w_m(x,\xi)\right)d\xi = 0.$$

So, the stationary conditions can be written as follows:

$$\lambda'(\xi) = 0, \quad 1 + \lambda(\xi)|_{\xi=t} = 0.$$

Thus, the Lagrange multiplier can be obtained as $\lambda = -1$, also and the following iteration formula is as follows:

$$w_{m+1} = w_m - \int_0^t \left(\frac{\partial w_m}{\partial \xi}(x,\xi) - (1+ia)\frac{\partial^2 w_m}{\partial x^2}(x,\xi) - Rw_m(x,\xi)\right.$$
$$\left. + (1+ib)w_m^{n+1}\bar{w}_m^{n}(x,\xi) + (1+id)w_m^{2n+1}\bar{w}_m^{2n}(x,\xi)\right)d\xi. \tag{18}$$

4. Sample examples

In this section, two generalized quintic complex Ginzburg–Landau equations are solved via the HAM based on Eqs. (15) and (16) and the results are shown in the tables. Also, the h-curves of each example are plotted. The programs and figures have been provided by Matlab package.

Example 1. Consider the following PDE:

$$w_t = (1+i)w_{xx} + 3w - (1+2i)|w|^2 w - (1-4i)|w|^4 w, \; w(x,0) = e^{ix}, \; i^2 = -1.$$

By the HAM, we get

$$w_0(x,t) = e^{ix},$$
$$w_1(x,t) = -hte^{xi}i,$$
$$w_2(x,t) = -(hte^{xi}(2hi + ht + 2i))/2,$$
$$w_3(x,t) = -(hte^{xi}(-h^2t^2i + 6h^2t + 6h^2i + 6ht + 12hi + 6i))/6, \ldots$$

setting h=-1, we obtain

$$w_0(x,t) = e^{ix},$$
$$w_1(x,t) = ite^{xi},$$
$$w_2(x,t) = -(t^2e^{xi})/2 = \frac{(it)^2 e^{ix}}{2!},$$
$$w_3(x,t) = -(t^3e^{xi}i)/6 = \frac{(it)^3 e^{ix}}{3!}, \ldots$$

so, we have

$$\begin{aligned} w(x,t) &= w_0(x,t) + w_1(x,t) + w_2(x,t) + \ldots \\ &= e^{ix} + \frac{(it)^2 e^{ix}}{2!} + \frac{(it)^3 e^{ix}}{3!} + \ldots \\ &= e^{ix}\left(1 + \frac{(it)^2}{2!} + \frac{(it)^3}{3!} + \cdots\right) = e^{i(x+t)}, \end{aligned}$$

which is the exact solution of this example.

Table 1 shows the errors of the HAM at $x = 1$ when $m = 15$ and $h = -1$, for different values of t. Also, Fig. 1 shows the h-curve of w_{tt} obtained from the 7-approximation via the proposed method.

Table 2 shows the convergence of the HAM at the point $(2, 0.7)$ and m = 4, 8, 12, 16, also the error column is calculated by $\left|\sum_{i=0}^{m} w_i - w\right|$. Fig. 2 shows the Imaginary and real parts of exact and approximation solutions of the proposed method, when $x = -1$ and different values of t, for Example 1.

Applying Eq. (18) as the variational iteration method, the following results are obtained:

$$w_0(x,t) = e^{ix}, \; w_1(x,t) = e^{ix} + ite^{xi}, \; w_2(x,t) = e^{ix} + ite^{xi} - (t^2e^{xi})/2, \ldots$$

so, it can be seen:

$$\lim_{m\to\infty} w_m = e^{ix}\left(1 + \frac{(it)^2}{2!} + \frac{(it)^3}{3!} + \cdots\right) = e^{i(x+t)}.$$

It means that, the VIM is also able to obtain the solution of the equation by a convergent recursive scheme and this result can be found via the HAM in the special case of $h = -1$.

Example 2. Consider the following PDE:

$$w_t = (1+3i)w_{xx} + 3w - (1+i)|w|^4 w - (1-6i)|w|^8 w, \; w(x,0) = e^{-ix}, \; i^2 = -1,$$

applying the HAM, we get

$$w_0(x,t) = e^{-ix},$$
$$w_1(x,t) = -2hte^{-xi}i,$$
$$w_2(x,t) = -2hte^{-xi}(hi + ht + i),$$
$$w_3(x,t) = -(4hte^{-xi})(-h^2t^2i + 3h^2t + (3h^2i)/2 + 3ht + 3hi + (3i)/2))/3, \ldots,$$

setting $h = -1$, we obtain

$$w_0(x,t) = e^{-ix},$$
$$w_1(x,t) = 2ite^{-xi},$$
$$w_2(x,t) = -2t^2e^{-xi} = \frac{(2it)^2e^{-ix}}{2!},$$
$$w_3(x,t) = -4(t^3e^{-xi}i)/3 = \frac{(2it)^3e^{ix}}{3!},\ldots,$$

so, we have

$$\begin{aligned} w(x,t) =& w_0(x,t) + w_1(x,t) + w_2(x,t) + \ldots \\ =& e^{-ix} + \frac{(2it)^2e^{-ix}}{2!} + \frac{(2it)^3e^{-ix}}{3!} + \ldots \\ =& e^{-ix}\left(1 + \frac{(2it)^2}{2!} + \frac{(2it)^3}{3!} + \ldots\right) = e^{i(2t-x)}, \end{aligned}$$

which is the exact solution of this example.

Fig. 3 presents the h-curve of for $w_t(1,0)$. Also, Figs. 4 and 5 compare the imaginary and real parts of the approximate and the exact solution, where $-2 \leqslant x \leqslant 2$.

5. Conclusion

In this paper, the homotopy analysis method was used to solve the generalized quintic complex Ginzburg–Landau equation. In order to show the importance and applicability of the proposed method, two examples by plotting of the h-curves and some related numerical results were thoroughly illustrated. Also, in an example, it can be observed that the VIM is also a proper and efficiency method to solve Eq. (10), whereas it is not able to control and adjust the region of convergence. Consequently, the HAM can be applied to solve the generalized quintic GCGL equation as a reliable and valid scheme.

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